

# Quadratic Effective Action for QED in $D = 2, 3$ Dimensions

D. Dalmazi\*

*Department of Physics and Astronomy - SUNY  
Stony Brook, NY 11794 , USA*

A. de Souza Dutra<sup>†</sup> and Marcelo Hott<sup>‡</sup>

*UNESP - Campus de Guaratinguetá - DFQ  
Av. Dr. Ariberto Pereira da Cunha, 333  
CEP 12500-000 - Guaratinguetá - SP - Brazil.*

December 30, 1999

## Abstract

We calculate the effective action for Quantum Electrodynamics (QED) in  $D = 2, 3$  dimensions at the quadratic approximation in the gauge fields. We analyse the analytic structure of the corresponding nonlocal boson propagators nonperturbatively in  $k/m$ . In two dimensions for any nonzero fermion mass, we end up with one massless pole for the gauge boson. We also calculate in  $D = 2$  the effective potential between two static charges separated by a distance  $L$  and find it to be a linearly increasing function of  $L$  in agreement with the bosonized theory (massive Sine-Gordon model). In three dimensions we find nonperturbatively in  $k/m$  one massive pole in the effective bosonic action leading to screening. Fitting the numerical results we derive a simple expression for the functional dependence of the boson mass upon the dimensionless parameter  $e^2/m$ .

*PACS-No.:* 11.15.Bt , 11.15.-q

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\*dalmazi@feg.unesp.br (On leave from UNESP- Guaratinguetá - Brazil ).

<sup>†</sup>dutra@feg.unesp.br

<sup>‡</sup>hott@feg.unesp.br

# 1 Introduction

The mapping of fermionic theories into bosonic ones is a very powerful technique used to understand nonperturbative aspects of quantum field theories. This so called bosonization is exact in two dimensional theories, and it has been widely employed in this dimension [1-3]. In the last few years, many papers have been devoted to the study of what has been called bosonization in three dimensions [4-8], and even in four dimensions [9]. This kind of path-integral bosonization consists of obtaining the effective action by integrating out the fermion degrees of freedom and then studying the physical properties of the resulting effective theory. In most of the works along this line one makes use of the derivative expansion [10, 11] and derives weak bosonization rules for the system [4, 6, 12] as well as the particle content and their masses [4]. On the other hand in [5] an exact, in  $k/m$ , action at one-loop level is used to show that the bosonization of [13] and [14], is recovered in the large and small momentum limits respectively. In addition, the authors of [12] have studied the asymptotic properties of the bosonic effective action associated with QED in three dimensions, showing the screening property of the effective potential between two static charges.

In this work we apply the effective action approach of bosonization on QED in two and three dimensions. The usefulness of the two dimensional case lies on the fact that  $QED_2$  can be bosonized via the massive Sine-Gordon model which exhibits confinement [1, 2, 3]. Thus both approaches of bosonization can be compared. Here we verify that, indeed the confining behavior also appears in the bosonization *via* effective action. It is remarkable that in two dimensions at the quadratic approximation in the gauge fields but without any expansion in  $k/m$ , the massive pole of the Schwinger model disappears, being replaced by a massless pole, which is in agreement with what has been observed in [15], but it differs from the result obtained through perturbative ( $m/e$ ) calculation of [16]. In three dimensions it is shown that there is a massive excitation which depends on the dimensionless parameter  $\frac{16\pi m}{e^2}$ . We have found a simple approximated expression for this function. This in fact generalizes the calculations of [4], which were obtained at leading order of the derivative expansion, and that of [17] carried out at a higher order in  $k/m$ , which in its turn is related to consistent higher derivative actions [18, 19].

In both cases we consider the one-loop effective action up to second-order in the coupling constant  $e$ . We may write it as

$$S_{eff}^{(2)} = -\frac{1}{2} \int \frac{d^D k}{(2\pi)^D} \tilde{A}_\mu(-k) \left[ g^{\mu\nu} k^2 - k^\mu k^\nu - \Pi^{\mu\nu}(k) \right] \tilde{A}_\nu(k), \quad (1)$$

where  $\tilde{A}_\mu(k)$  is the Fourier transformation of  $A_\mu(x)$  and

$$\Pi^{\mu\nu}(k) = ie^2 \int \frac{d^D p}{(2\pi)^D} \text{tr} \left[ \frac{1}{\not{p} - m + i\epsilon} \gamma^\mu \frac{1}{(\not{p} + \not{k}) - m + i\epsilon} \gamma^\nu \right] \quad (2)$$

is the polarization tensor obtained after integrating out the fermion fields. The space-time dimension is represented by  $D$  ( $D = 2, 3$ ). It is useful to expand the polarization tensor in powers of  $\frac{k}{m}$  which corresponds to the derivative expansion of the effective action [20]. Truncating the expansion at some power of  $k/m$  not only yields a local effective action but also allows one to analyse the rôle of some specific term as it is the case of the leading odd-parity term in  $QED_3$  [21]. It is also important to study the phenomenological aspects of a low-energy effective action [22]. The order at which the series is truncated depends on the range of energy one is interested in.

Recent studies of the contributions of higher-order derivative terms in a low-energy effective gauge theory revealed the possibility of the appearance of non-physical excitations. Here we overcome this difficulty by analyzing directly the poles of the full propagator at one-loop level up to second order in the coupling constant.

## 2 Effective potential in $QED_2$

In  $QED_2$  the polarization tensor will be given by

$$\Pi^{\mu\nu}(k) = \Pi(k^2) \left( g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right), \quad (3)$$

where using , e. g. , dimensional regularisation we get

$$\Pi(k^2) = \frac{e^2}{\pi} \left[ 1 + \frac{1}{2} \frac{4m^2/k^2}{(1 - 4m^2/k^2)^{1/2}} \ln \frac{(1 - 4m^2/k^2)^{1/2} + 1}{(1 - 4m^2/k^2)^{1/2} - 1} \right]. \quad (4)$$

We are interested in the interacting potential between two static charges  $Q$  and  $-Q$  located at  $x = L/2$  and  $x = -L/2$ . Solving<sup>1</sup> the equation of motion derived from the effective action we obtain the potential produced by the positive charge  $Q$  :

$$A_\mu(x) = \int \frac{d^2k}{(2\pi)^2} \int d^2x' D_{\mu\nu}(k) e^{ik(x-x')} J^\nu(x'), \quad (5)$$

where

$$J^\nu(x') = Q \delta(x_1' - \frac{L}{2}) \delta^{\nu 0} \quad (6)$$

is the conserved current and  $D_{\mu\nu}(k)$  is the “photon” propagator whose longitudinal term is given by

$$D^\parallel_{\mu\nu}(k) = \frac{1}{k^2 - \Pi(k^2)} g_{\mu\nu}. \quad (7)$$

Due to the specific form of the external current the only contribution to the potential will be its time-component . The corresponding integral can be easily performed on the complex plane for arbitrary masses . The resulting static potential at  $x_1 = -\frac{L}{2}$  grows linearly with the distance between the charges, namely

$$A_0(x = -\frac{L}{2}) = \frac{Q}{2 \left[ 1 + \frac{2}{3\pi} (\frac{e}{2m})^2 \right]} L. \quad (8)$$

Then we conclude that, at the quadratic approximation used here,  $QED_2$  results in a *confining* potential. Because of the quadratic approximation in the gauge fields we may say that the result obtained here, *i.e.*, a linearly growing inter-fermion effective potential is due to a zero mass pole for the gauge potential. One also reaches a similar result in the usual bosonization approach when the quadratic approximation in the boson field of the massive Sine-Gordon model is used [15]. In other words we can say that, at the quadratic approximation, turning on a mass for the fermion field will prevent the mass generation for the gauge boson and the classical result, that is a massless pole, prevails. It is worth mentioning that , since we are only

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<sup>1</sup>We have not taken into account the general solution of the homogeneous differential equation ( without sources ) which amounts to neglect the  $\theta$ -vacuum ( $\theta = 0$ )

interested in the real contribution to the effective action , we have dropped in (4) the imaginary part of the polarization tensor which appears beyond the pair creation threshold  $k^2 > 4m^2$ . More specifically , expression (4) is correct for  $k^2 < 0$  which is the relevant region for the calculation of the effective potential of static charges due to the factor  $\delta(k_0)$  which comes from the time integral in (5). However, for the analysis of the particle content of (1) we have also calculated the polarization tensor in the region  $0 < k^2 < 4m^2$  where we found no poles except in the limit  $k^2 \rightarrow 0^+$  where we found an agreement with the limit  $k^2 \rightarrow 0^-$  of (4). Finally, it is important to notice that although we can recover the Schwinger model effective action from the  $m \rightarrow 0$  limit of (4) its static potential does not correspond to the massless limit of (8) since the integral and the limit do not commute with each other. In the Schwinger model (massless  $QED_2$ ) a mass for the gauge boson is generated by the gauge anomaly and as a consequence one obtains a *screening* potential between two static charges [23, 24].

### 3 Massive pole in $QED_3$

Here we are not concerned with the explicit expression for the interaction potential but with the behavior of the mass generated dynamically as a function of the coupling constant and the fermion mass.

In this case the polarization tensor is given by

$$\Pi^{\mu\nu}(k) = \Pi_1(k^2)i\epsilon^{\mu\nu\rho}k_\rho + \Pi_2(k^2)\left(k^2 g^{\mu\nu} - k^\mu k^\nu\right), \quad (9)$$

where

$$\Pi_1(k^2) = -\frac{e^2}{8\pi} \frac{1}{\sqrt{k^2/4m^2}} \ln \frac{1 + \sqrt{k^2/4m^2}}{1 - \sqrt{k^2/4m^2}}, \quad (10)$$

and

$$\Pi_2(k^2) = \frac{e^2}{16\pi m} \frac{1}{k^2/4m^2} \left[ 1 - \frac{1}{2} \left( \frac{1 + k^2/4m^2}{\sqrt{k^2/4m^2}} \right) \ln \frac{1 + \sqrt{k^2/4m^2}}{1 - \sqrt{k^2/4m^2}} \right]. \quad (11)$$

Equations (10) and (11) were obtained by using dimensional regularization method. Equation (10) is regularization dependent and our result

corresponds to an equal number of Pauli-Villars regulators with positive and negative masses. Like the two dimensional case , since we are only interested in the real contribution to the effective action, we have dropped the imaginary part of the polarization tensor which appears beyond the pair creation threshold (  $k^2 > 4m^2$  ). More precisely we have only given the polarization tensor (see (10) and (11)) in the region  $0 < k^2 < 4m^2$  where we found a pole in the photon propagator. For  $k^2 < 0$  the correct result correspond to replace  $\frac{1}{\sqrt{u}} \log((1+\sqrt{u})/(1-\sqrt{u}))$  by  $\frac{-2}{\sqrt{-u}} \arctan \frac{1}{\sqrt{-u}}$  in (10) and (11) where  $u = \frac{k^2}{4m^2}$ . The situation is similar to QED<sub>4</sub> , see , e.g., [26]. We have explicitly verified that no tachyonic poles appear without any approximation on  $\frac{k^2}{4m^2}$  . Back to the region  $0 < k^2 < 4m^2$  one can check that the “photon” propagator

$$D^{\parallel}_{\mu\nu}(k) = -\frac{1}{\tilde{\Pi}_2 \left[ k^2 - \left( \frac{\Pi_1}{\Pi_2} \right)^2 \right]} g_{\mu\nu}, \quad (12)$$

where

$$\tilde{\Pi}_2(k) = 1 - \Pi_2(k),$$

develops a massive pole  $k^2 = M^2$ . The gauge boson mass  $M$  depends on the coupling constant and fermion mass. We have carried out a numerical analysis for the behavior of this mass generated dynamically and found a very simple function , namely

$$M = \frac{2m}{c_1 + c_2 a + c_3 a^2}, \quad (13)$$

where

$$a = 16\pi \frac{m}{e^2}. \quad (14)$$

From the numerical results for the massive pole at small  $a$  we get  $c_1 = 1.953331381$  . On the other hand , from the large  $a$  region we have  $c_3 = 1$ . Finally the constant  $c_2$  was adjusted by choosing the best fitting for a curve with about eleven thousand numerically calculated points, and it was found to be  $c_2 = 2.253$ . The maximum error of the fitted function is less than 3.8%, with the statistical controlling parameter  $\chi^2 = 5.5 \times 10^{-6}$ . In the figure 1 we present the curve of the massive pole as a function of the dimensionless parameter  $a$ . Note that it is impossible to distinguish the exact from the adjusted curve in figure 1. For this reason we present in the

figure 2 the dependence of the percentual deviation of the fitted curve from the numerically obtained masses.

In reference [12] the derivative expansion up to second-order in  $\frac{k}{m}$  is used to compute the effective potential which was also found to be of the *screening* type. We have taken into account the whole expression for the polarization tensor and found only one massive pole in the whole range of the parameter  $a$ . Since the pole can never be found at the origin we conclude that, non-perturbatively in  $k/m$  the screening effect prevails in agreement with the numerical analysis carried out in reference [25] for the static potential. The truncation based on the derivative expansion only amounts to a displacement of the massive pole from its nonperturbative ( in  $k/m$  ) value.

## 4 Acknowledgements

We would like to thank professors E. Abdalla , C. de Calan and J. A. Mignaco for valuable discussions. This work was partially supported by CNPq and FAPESP.

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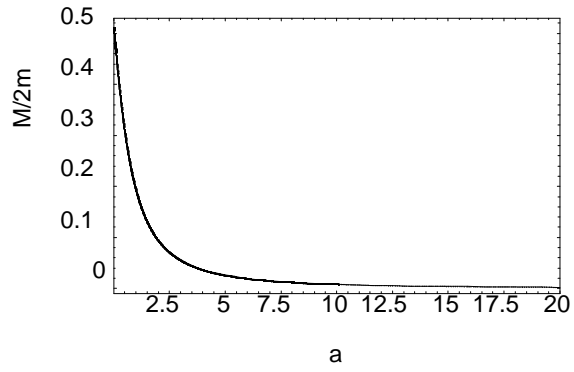


Figure 1: Dependence of the rate of the massive bosonic pole and fermion mass with respect to the inverse coupling  $a = 16\pi \frac{m}{e^2}$ .

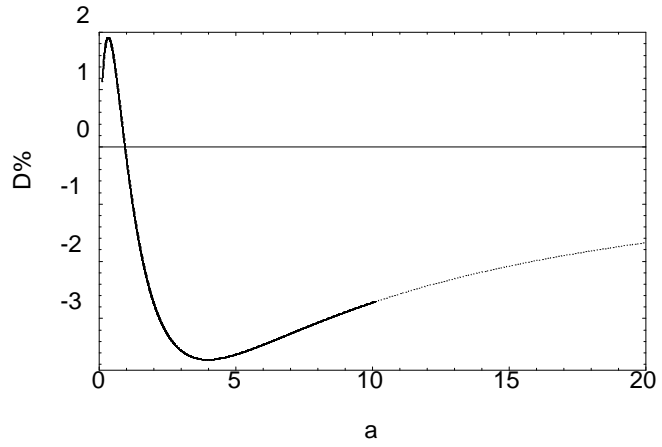


Figure 2: Percentual difference between the fitting and the numerical data